Fisheries objectives

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Fisheries objectives

- How much should we catch considering
  - Stock growth
  - Costs of fishing
  - Discount rate
- Methods
  - Graphical analysis
  - Lagrange optimization
  - Optimal control (Maximum Principle)

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Graphical analysis

- But graphical analysis is quite limited
  - Stock-dependent catch (search fish)
  - Zero discount rate
- Algebraic analysis allows more general insights
  - Stock-independent catch (schooling fish)
  - Effect of discount rate

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The general problem

- Maximize the objective:

$$\pi = \int_0^T (pY(X,E) - C(Y,X,t))e^{-\rho t}dt$$

- With state equation:

$$\dot{X} = G(X) - Y(X,E)$$

- Stock variable:
  - X
- Control variable:
  - Y? E?

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The control variable in the fishery problem

- Catch and effort are interdependent through the catch function:
  $Y = qEX \Rightarrow E = \frac{Y}{qX}$
- If catch is the control variable:

Maximize $\pi = \int_0^T \left(pY - c\frac{Y}{qX}\right)e^{-\rho t}dt$ such that $\dot{X} = rX\left(1 - \frac{X}{K}\right) - Y$

- If effort is the control variable:

Maximize $\pi = \int_0^T (pqEX - cE)e^{-\rho t}dt$ such that $\dot{X} = rX\left(1 - \frac{X}{K}\right) - qEX$

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The control variable in the fishery problem

- The control variable can be either Y or E
- In implicit analyses (lecture slides)
  - Y is the control variable
  - Because it gives more insight
- In explicit analyses (numerical examples)
  - E is the control variable
  - Because it is easier

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Simplest case: schooling fish, no discounting

- Harvesting costs independent of stock size
- Control variable: Y (for didactic reasons)
- Zero discount rate
- We can therefore impose stability and use Lagrange:

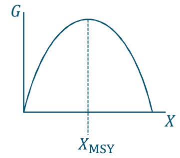
$$L = pY - C(Y) + \lambda(G(X) - Y) \Rightarrow (p - C_Y)G_X = 0$$

Marginal net benefits of catch

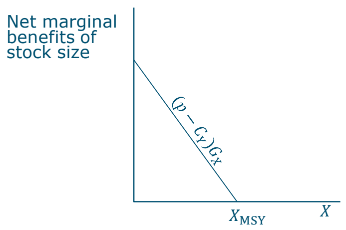
Marginal growth of stock size

Simplest case: schooling fish, no discounting

- So either the species is useless...
- ...or marginal biological growth is zero
  - Which corresponds to MSY



Simplest case: schooling fish, no discounting



Numerical example

- Consider a fishery with the following characteristics:

$p = 10$

$Y = 10E$

$C = E$

$G = 0.5X\left(1 - \frac{X}{40}\right)$

- Use E as the control variable (this is easier)
- What is the optimal fishing effort (E)?
- What is the optimal stock size (X)?

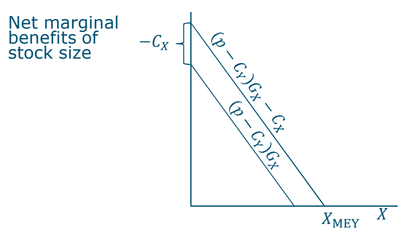
Extension: search fish (no discounting)

- Harvesting costs depends on stock size
  - Less abundant fish -> more difficult to catch
- Control variable: Y (it's an implicit model after all)
- Zero discount rate
- We can therefore impose stability and use Lagrange:

$$L = pY - C(Y, X) + \lambda(G(X) - Y) \Rightarrow (p - C_Y)G_X - C_X = 0$$

Cost savings from larger stock

Simplest case: schooling fish, no discounting



Effect of stock dependence of costs

- If costs are stock independent
  - Fish at MSY
- If costs are stock dependent
  - Fish at MEY
  - Larger stock size to reduce harvesting costs

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Numerical example

- Consider a fishery with the following characteristics:

$p = 1$
$Y = 0.2EX$
$C = 4E$
$G = 2X\left(1 - \frac{X}{50}\right)$

- Take E as the control variable (this is easier)
- What is the optimal fishing effort (E)?
- What is the optimal stock size (X)?

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Extension: discount rate

- Schooling fish: costs independent of stock size
- Control variable: Y (for didactic reasons)
- Positive discount rate
- Now we need to use optimal control
  - Present value Hamiltonian:

    $H = \left(pY - C(Y)\right)e^{-\rho t} + \lambda(G(X) - Y)$
  - Current value Hamiltonian:

    $H^c = \left(pY - C(Y)\right) + \mu(G(X) - Y)$

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Extension: discount rate

- For schooling fish the general rule is that marginal growth should equal stock:

  $G_X = \rho$
- Interpretation: two bank accounts
  - Financial: fixed interest rate
  - Biological: interest rate depends on stock size
- Where to put your money?
  - On the account with the highest interest rate

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Numerical example

- Consider a fishery with the following characteristics:

$p = 10$
$Y = 10E$
$C = E$
$G = 0.5X\left(1 - \frac{X}{40}\right)$
$\rho = 0.05$

- Take E as the control variable (it's easier)
- What is the optimal fishing effort (E)?
- What is the optimal stock size (X)?

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Alternative approach in discrete time

- Suppose you start with some pristine stock X
- Trade-off between two periods of time
  - Period 1 (one year): Catch amount Y
  - Period 2 (forever): Catch $G(X - Y)$

Period	1	2		
Year	1	2	3	Etc.
Stock in start of period	X	$X - Y + G(X - Y)$	$X - Y + G(X - Y)$	Etc.
Catch	Y	$G(X - Y)$	$G(X - Y)$	Etc.
Stock at end of period	$X - Y$	$X - Y$	$X - Y$	Etc.

- How large should Y be?

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Alternative approach in discrete time

- We face the following maximization problem:

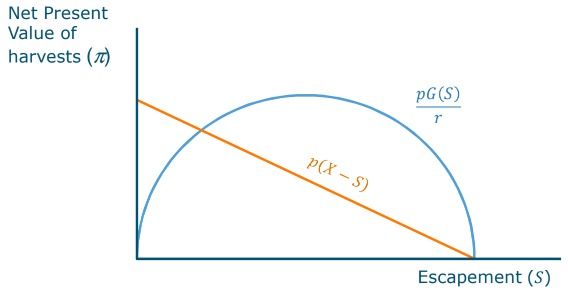
$$\max_{H} \pi = pY + \frac{pG(X-Y)}{r}$$

- In fact it is easier to solve like this:

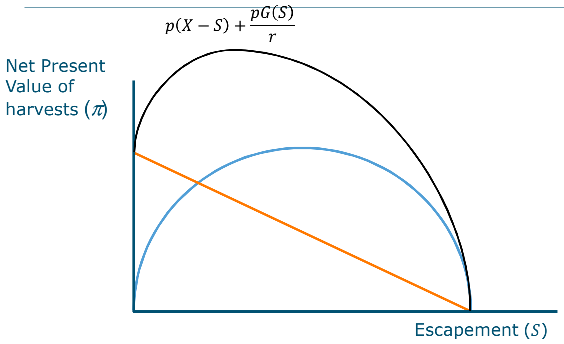
$$\max_{S} \pi = p(X-S) + \frac{pG(S)}{r}$$

- Where S is called 'escapement'

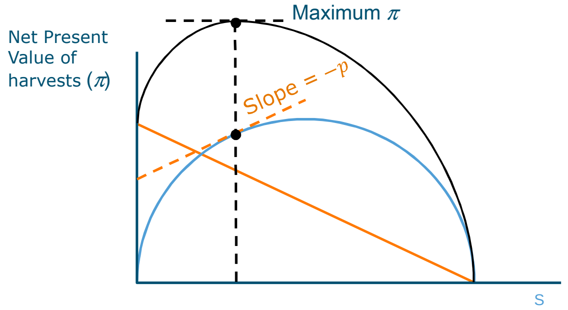
Alternative approach in discrete time



Alternative approach in discrete time



Alternative approach in discrete time



Alternative approach in discrete time

Take the first derivative with respect to escapement:

$$\pi = p(X-S) + \frac{pG(S)}{r} \Rightarrow \frac{d\pi}{dS} = -p + \frac{{dG}/{dS}}{r}p$$

- Increasing S by one fish has two effects:
  - In the present you forgo 1 fish times its price
  - In the future you earn the offspring of that fish times its price, annually
- In the optimum the two effects are equal:

$$p = \frac{{dG}/{dS}}{r}p \Rightarrow \frac{dG}{dS} = r$$

Extension: search fish and discount rate

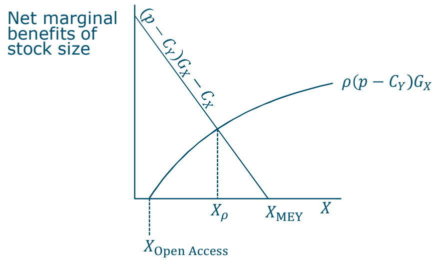
- Harvesting costs depends on stock size
  - Less abundant fish -> more difficult to catch
- Control variable: Y (it's an implicit model after all)
- Positive discount rate
- Now we need to use optimal control
  - Present value Hamiltonian:
    $H = \left(pY - C(Y,X)\right)e^{-\rho t} + \lambda\left(G(X) - Y\right)$
  - Current value Hamiltonian:
    $H^c = \left(pY - C(Y,X)\right) + \mu\left(G(X) - Y\right)$

Extension: search fish and discount rate

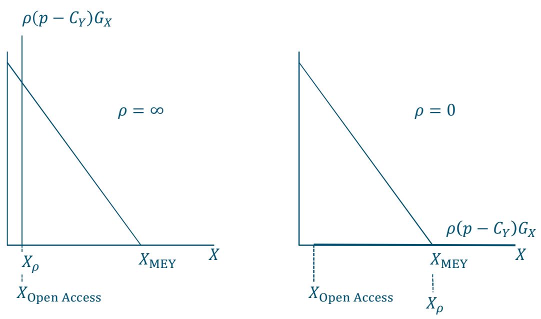
- Optimality condition:

$$\underbrace{(p - C_Y)G_X - C_X}_{\text{Marginal benefits of stock size (i.e., of postponing yield)}} = \underbrace{\rho(p - C_Y)}_{\text{Marginal benefits of harvesting now}}$$

Extension: search fish and discount rate



Extreme cases



Role of the discount rate

- At a zero interest rate you should fish at MSY or MEY
  - The future is just as important as the present
- As interest rate increases
  - The future becomes 'less important'
  - More exploitation
- At extremely high interest rates
  - It is more profitable to deplete the resource and put the money on the bank
  - Equivalent to open-access fishing

Comparing fish stocks and bank accounts

- Market interest rate:
  - Mostly between 0 and 20%
  - Currently about 1%

Species	r (Source:fishbase.org)
Atlantic herring	0.15 - 0.5
Atlantic cod	0.15 - 0.5
Plaice	0.05 - 0.15
Monkfish	0.05 - 0.15
Orange roughy	< 0.05

Next

Tomorrow:

13:30 – 15:15 in PC516

Practical fisheries models in R

15:30 – 17:15 in C79

Dynamic programming